$$\underset{a_k}{\underset{\|}{2^{n-3}}} = (\frac{1}{2})\cdot 2\cdot 2^{n-3} = \underset{a_1\cdot r^{k-1}}{\underset{\|}{(\frac{1}{2})\cdot 2^{n-2}}} \Rightarrow n-2=k-1 \Rightarrow k=n-1 \Rightarrow \text{第}\,n-1\,\text{項}$$

(2) 設 4^{n+1} 為第七項 $\underset{a_t}{\underset{\|}{4^{n+1}}} = (2^2)^{n+1} = 2^{2n+2} = \frac{1}{2}\cdot 2\cdot 2^{2n+2} = \underset{a_1\cdot r^{t-1}}{\underset{\|}{\frac{1}{2}\cdot 2^{2n+3}}}$

$\Rightarrow 2n+3=t-1 \Rightarrow t=2n+4 \Rightarrow$ 第 $2n+4$ 項

主題４　等比級數

[觀念一] 等比級數求和

公式：等比級數的一般項 $a_{n+1}=a_n r$ ，$n=1$，2，3，…，

$$a_1+a_2+a_3+\cdots+a_n$$
$$=a_1+a_1r+a_1r^2+\cdots+a_1r^{n-1}$$
$$=\begin{cases} na_1 & \text{當 } r=1 \\ \dfrac{\overset{\text{首項}}{a_1}(1-r^{\overset{\text{項數}}{n}})}{1-\underset{\text{公比}}{r}} & \text{當 } r\neq 1 \end{cases}$$

[範例一]

(1) $9+99+999+\cdots+$第 n 項=________　(2) $0.9+0.99+0.999+\cdots+$第 n 項=________

答 (1) $\frac{1}{9}(10^{n+1}-9n-10)$　(2) $\frac{1}{9}(9n-1+\frac{1}{10^n})$

解 (1) $9+99+999+\cdots+$第 n 項

$$=(10-1)+(10^2-1)+(10^3-1)+\cdots+(10^n-1)$$
$$=(10+10^2+10^3+\cdots+10^n)-n$$
$$=\frac{10(1-10^n)}{1-10}-n=\frac{10(1-10^n)}{-9}-n=\frac{1}{9}(10^{n+1}-9n-10)$$

(2) $0.9+0.99+0.999+\cdots+$第 n 項

$$=(1-\frac{1}{10})+(1-\frac{1}{10^2})+(1-\frac{1}{10^3})+\cdots+(1-\frac{1}{10^n})$$
$$=n-(\frac{1}{10}+\frac{1}{10^2}+\cdots+\frac{1}{10^n})$$
$$=n-\frac{\frac{1}{10}(1-\frac{1}{10^n})}{1-\frac{1}{10}}=n-\frac{(1-\frac{1}{10^n})}{9}=\frac{1}{9}(9n-1+\frac{1}{10^n})$$

[範例二]

(1) $1+2+4+\cdots+2^n=$________　(2) $\frac{1}{4}+\frac{1}{2}+1+2+4+\cdots+2^{n-5}=$________